

INCOME TAXATION AND GROWTH IN AN OLG ECONOMY: DOES AGGREGATE UNCERTAINTY PLAY ANY ROLE?*

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We analyze the effects of capital income taxation on long-run growth in a stochastic, two-period overlapping generations economy with an aggregate AK technology. We distinguish between capital income and labor income, and between attitudes towards risk and intertemporal substitution of consumption. We show necessary and sufficient conditions such that i) increments in the capital income taxation lead to higher equilibrium growth rates, and ii) the effect of changes in the capital income tax rate on the equilibrium growth may be of opposite signs in stochastic and in deterministic economies. Numerical simulations show that deterministic OLG economies are a good approximation of the assessment of the effect of taxes on the equilibrium growth rate in stochastic economies as in Smith (1996).

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The relative performance of capital and labor income taxations is the subject of a long-standing debate in the economic literature in which two main strands can be distinguished. The first one includes those papers that analyze the optimality of taxing capital income. Chamley (1986) was the first to prove analytically the optimality of a zero capital income tax rate as time tends to infinity. The result also applies to finite life-time horizons (provided there exists intergenerational altruism à la Barro) and to the case in which leisure enters the utility functions of individuals. Atkeson *et al.* (1999) show that the result also continues to hold in finite horizon overlapping generations economies with no growth and under conditions of homotheticity and separability in the utility function. In particular, they show that, if intertemporal elasticity of substitution (IES in the sequel) is higher than one, then the optimal capital income tax is zero¹. Atke-

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(1) The opposite result could also be obtained, however, if the economy is populated by individuals with a sufficiently low IES.

son *et al.* (1999) prove that the result continues to hold even when the assumptions in the model are sequentially relaxed to allow for heterogeneous agents (but the government is allowed to tax different individuals in a different way), open economies and endogenous growth through physical and human capital accumulation (in which case all taxes have to be zero). Lucas (1990) shows the optimality of zero capital income tax rate in an endogenous growth model in which individuals are infinitely-lived and growth is driven by human capital accumulation. If the transition between steady states were also considered and the government were allowed to run short-term surpluses or deficits, Atkeson *et al.* (1999) show that the optimal physical capital income tax rate should tend to zero after a finite number of periods. In recent years, however, some studies show that taxing capital income might not be optimal [see, among others, Conesa *et al.* (2009)²].

The second strand comprises those papers that analyze the growth effects of taxing capital income. Focusing on infinitely-lived agents' economies, Lucas (1990) finds a negative effect of capital income taxation on the balanced growth path. This result is the natural one in AK technology economies [see, e.g., Jones and Manuelli (2005)]. As for OLG model economies, Uhlig and Yanagawa (1996) find, in a two-period OLG deterministic setup, with an AK technology, that an increase in the physical capital income tax may give rise to higher growth rates. This will be so provided that the young individual's saving elasticity with respect to the interest rate is low enough, which can be guaranteed if IES is low enough. Caballé (1998), in a two-period OLG model economy with an AK technology, finds a threshold value for the bequest motive to be operative. In this setup, the capital income tax rate that maximizes growth can be either zero (if the bequest motive is operative and the economy behaves as an infinitely-lived agent economy) or one (otherwise and preferences are characterized by a low enough IES). Smith (1996), however, in an infinitely-lived agent economy setup, finds that the growth-maximizing capital income tax rate may be positive in the presence of aggregate uncertainty. This is so, because increments in the capital income tax rate imply lower risk of the return to capital and their final effect on households' savings will depend on their IES³.

In this paper we take up the second type of issue. We analyze the effect of capital income taxation on long run growth in a model economy with aggregate uncertainty and an AK technology, and where the economy is populated by generations whose individuals live for two periods and whose labor supply is inelastic, making an explicit distinction between the IES and risk aversion. We assume that the government balances its budget on a period basis and that any change in the capital income

(2) As claimed by Conesa *et al.* (2009), one can find at least two setups in which the long-run zero capital income tax result may fail to hold: i) if households face binding borrowing constraints and/or are subject to uninsurable idiosyncratic income risk; and ii) in lifecycle models in which the tax code cannot explicitly be conditioned on the age of the household.

(3) Jones and Manuelli (2005) present an excellent review of the effects of fiscal policies and fluctuations (aggregate uncertainty) on growth in economies with infinitely-lived agents. In particular, they remark the ambiguous effect of uncertainty on growth depending on the households' IES. They do not distinguish, however, between intertemporal elasticity of substitution and risk aversion, as in Smith (1996).

tax rate will be accompanied by a change in labor income tax rate⁴. Growth is driven by the accumulation of young individuals' savings in the form of physical capital. To the extent that income taxation not only affects current and future net incomes, but also the risk of future net income, a higher capital income tax rate may exert a different effect on savings (and growth) in stochastic economies with respect to deterministic economies. More precisely, we obtain conditions under which the total effect of the capital income tax on the equilibrium growth rate might have a different sign depending on whether the economy is stochastic or deterministic.

Consider first the case of a deterministic economy. An increase in the capital income taxation (and, consequently, a decrease in the labor income taxation) will imply i) higher savings due to the increase in current labor income, ii) lower savings due to the increase in future labor income, and iii) depending on the IES (or, in other words, the interest rate elasticity), higher savings if and only if $IES < 1$. Therefore, if (say) $IES < 1$, it is more likely that the growth rate increases as the capital income tax rate is increased. Consider now a stochastic economy: a higher capital income tax rate leads to a reduction in the riskiness of future capital income and an increment of that of future labor income, so that the respective sizes of those effects change and, in turn, the sign of the total effect can also change.

The two most closely related papers to ours are Smith (1996) and Uhlig and Yanagawa (1996). Our paper departs from them in two key aspects. It differs from Smith (1996) in that we consider infinite generations of finitely-lived individuals. Were the economy in Smith (1996) deterministic, higher (capital) income taxation would always (i.e. regardless of the IES) produce lower growth rates. Under uncertainty, however, and as stated above, higher capital income taxation may induce higher equilibrium growth rates. Our paper differs from Uhlig and Yanagawa (1996) in that we introduce aggregate uncertainty, and distinguish between attitudes towards i) time preference and ii) risk. Here, as already remarked by Uhlig and Yanagawa (1996) in a deterministic setup, the effect of capital income taxation on savings will depend on individuals' IES. We find conditions under which the capital income tax policy will exert different sign effects on equilibrium growth in stochastic and deterministic economies. Thus, we find that a sign reversal of the growth effects of the capital income taxation policy considered in Uhlig and Yanagawa (1996) is possible.

A summary of the main results follows:

1. We break down the total effect of capital income taxation on the equilibrium growth rate into three partial effects on i) current net-of-tax labor income, ii) return to savings, and iii) future net-of-tax labor income. We focus on the role played by uncertainty in evaluating these partial effects (and, ultimately, the total effect) of capital income taxation on growth.

2. We obtain necessary and sufficient conditions for increments in capital income taxation to induce higher equilibrium growth rates in stochastic and in deterministic economies.

(4) Alternative tax policies will be discussed in section 3.3.

3. Assuming that households' second-period labor supply is zero, we obtain a necessary and sufficient condition implying that the IES is higher than one and that increments in the capital income tax rate will lead to higher steady-state growth rates in deterministic economies, but lower in stochastic economies. If the IES is less than one, however, increases in the capital income tax will always induce higher equilibrium growth rates, both in stochastic and non-stochastic economies.

4. If individuals' second-period labor supply is positive, we obtain two necessary and sufficient conditions for increments in the capital income tax rate to have opposite sign effects on the equilibrium growth rate in stochastic and in deterministic economies depending on how IES compares to one. If IES is less (resp. higher) than one, increments in the capital income taxation may generate higher growth rates in stochastic economies, but lower in deterministic ones (resp. lower growth rates in stochastic economies, but higher in deterministic ones).

5. We show analytically that no sign reversal can be obtained under two alternative tax policies.

6. We build a numerical example after calibrating our model economy to mimic some stylized facts of the U.S. economy to illustrate some of the results. We show the critical roles played by both the IES and the second-period labor supply in obtaining that sign reversion. For reasonable values of the IES (i.e. less than 1), a sign reversal only appears for unreasonably high values of the second-period labor supply. For values of the IES > 1 , however, reversions of the sign arise for all second-period labor supply values considered. The conclusion is a neat one: deterministic OLG economies present a good approximation to the effects of taxes on growth as in Smith (1996).

The rest of the paper is organized as follows. Section 1 describes the economy. Section 2 solves the equilibrium growth rate. Section 3 analyzes its properties in terms of its response to income tax policy. Section 4 provides numerical results. Section 5 concludes. A formal appendix provides the mathematical proofs.

1. THE ECONOMY

The model draws heavily on Uhlig and Yanagawa (1996) and Weil (1990)⁵. There are two sectors in the economy: a private one (households and firms) which makes its decisions in a perfectly competitive market framework, and a government which levies a proportional income tax (that taxes labor and capital income at different rates) to finance some exogenous level of expenditure which is neither productive nor enters households' preferences.

As for the households, the economy is populated by a continuum of young individuals and a continuum of old individuals which coexist at any time, both growing at an exogenous, constant rate, n . The productive sector is represented by a continuum of competitive firms of measure one. All firms use the same production technology of constant returns to scale in capital and labor, and are exposed

(5) The model here presented is similar to that of "family businesses" in Mauro (1995) except for the existence of government sector and income tax policy.

to a positive externality given by the aggregate stock of capital per unit of labor. The appropriate choice of parameters will make firms exhibit an aggregate AK technology in equilibrium, thereby allowing for the existence of sustained growth. Therefore, capital accumulation is the source of endogenous growth. Furthermore, all firms are exposed to the same aggregate technological shock, thus introducing uncertainty in the economy.

1.1. Households

Suppose an individual born at time t who lives for 2 periods and whose preferences over young and old period consumption ($c_{1,t}$ and $c_{2,t+1}$, respectively) are of the class introduced by Weil (1990)⁶. In particular, he/she maximizes the utility function

$$V_t(c_{1,t}, c_{2,t+1}) = \frac{c_{1,t}^{1-(1/\sigma)} - 1}{1-(1/\sigma)} + \beta \frac{\left\{ \left[E_t c_{2,t+1}^{1-\eta} \right]^{1/\sigma} \right\}^{1-(1/\sigma)} - 1}{1-(1/\sigma)}, \quad [1]$$

where $\sigma > 0$, $\sigma \neq 1$ denotes the IES, $\eta > 0$ denotes the constant relative risk aversion coefficient, $\beta \in (0,1)$ denotes a time preference parameter which, in the case of no uncertainty, denotes the discount factor, $E_t(\cdot)$ denotes the rational expectation

operator conditional on the information set available as of time t , and $\left[E_t c_{2,t+1}^{1-\eta} \right]^{1/\sigma}$ is the certainty-equivalent second period consumption. Note that i) if the economy were deterministic, or individuals had perfect foresight or were risk neutral, risk aversion would play no role; and ii) if $\eta > 1/\sigma$, we would obtain the standard expected utility case⁷. In the latter case, however, we would be making no distinction between attitudes to risk and to intertemporal substitution. As Kimball and Weil (2009) pointed out “(...) the traditional theory of precautionary saving based on intertemporal expected utility maximization is a framework within which one cannot ask questions that are fundamental to the understanding of consumption [savings] in the face of labor income risk”. [See Kimball and Weil (2009), p. 245]. Therefore, the need to make such a distinction justifies the use of a utility function such as the one in Eq. [1].

Individuals inelastically supply 1 unit of labor in their first period and a fraction $\theta \in (0,1)$ in their second period⁸. The relevance of the role played by θ will become apparent in the discussion of the results: note that the higher θ , the lower the first-period savings. Denoting first-period savings by s_t , the net-of-tax wage rate per unit of labor at t by w_t^n , and the net-of-tax interest rate between t and $t + 1$ by w_{t+1}^n , the individual’s first and second period budget constraints become

$$c_{1,t} = w_t^n - s_t, \quad [2]$$

(6) The first subscript denotes age (1, young; 2, adult) and the second subscript denotes calendar time.

(7) See, e.g. Backus, Roulledge and Zin (2004-2008) and references therein for a thorough introduction to recursive and ordinal certainty-equivalent preferences.

(8) While restrictive, the inelastic labor supply assumption is needed to let the model admit analytical solution.

and

$$c_{2,t+1} = (1 + r_{t+1}^n) s_t + \theta w_{t+1}^n, \quad [3]$$

respectively. Assuming proportional income taxation and denoting labor and capital income tax rates by τ_t^w and τ_t^k , respectively, net-of-tax factor prices are given by $w_t^n \equiv (1 - \tau_t^w) w_t$ and $r_t^n \equiv (1 - \tau_t^k) (1 + r_t) - 1$. Here, as in Uhlig and Yanagawa (1996), we are assuming that capital taxes are paid on the full amount of income: the resale value of the capital, plus the capital gains⁹.

Therefore, substituting $c_{1,t}$ and $c_{2,t+1}$ from Eqs. [2] and [3], respectively, into the objective function in Eq. [1], and maximizing the resulting equation with respect to s_t yields the following first order necessary (and sufficient) condition

$$c_{1,t}^{-1/\sigma} = \beta \left[\left(E_t c_{2,t+1}^{1-\eta} \right)^{\frac{1}{1-\eta}} \right]^{\eta-(1/\sigma)} E_t \left[(1 + r_{t+1}^n) c_{2,t+1}^{-\eta} \right] \quad [4]$$

Eq. [4] represents the stochastic Euler equation, which is a crucial one since the engine of economic growth is capital accumulation which, in turn, depends only on the first period savings. Given that the tax policy affects not only current and future disposable income, but also uncertainty about the latter, it will also influence the growth rate. Note that a higher capital income tax rate implies not only lower expectation, but also lower variance of future net return to savings, as in Smith (1996). Furthermore, if $\theta < 0$, certainty-equivalent second period consumption will also depend on both the expectation and the variance of future net labor income. And if a higher capital income tax rate is accompanied by a lower labor income tax rate, increased capital income taxation will have opposite sign effects on the variance (risk) of both types of future income: capital and labor income.

1.2. Firms

Let us suppose that a firm acts competitively in the output and production factor markets without adjustment costs in production inputs¹⁰. Formally, the problem this firm faces is written as

$$\begin{aligned} \max_{\{Y_t^i, N_t^i, K_t^i\}} & Y_t^i - w_t N_t^i - (r_t + \delta) K_t^i \\ \text{s.t.} & Y_t^i = A_t (K_t^i)^\alpha (N_t^i)^{1-\alpha} k_t^{1-\alpha}, \end{aligned} \quad [5]$$

where Y_t^i denotes output, N_t^i denotes labor, K_t^i denotes physical capital, $\alpha \in (0,1)$ denotes the capital income share, and $\delta \in (0,1)$ denotes the physical capital depreciation rate. Some remarks concerning production technology follow. First, we assume that all firms (uniformly distributed on the interval [0,1]) are exposed to a

(9) While this assumption does not imply qualitative changes, it will allow us to obtain the analytical solution for the equilibrium growth rate.

(10) This follows Jones and Manuelli (1997) which, in turn, follows Romer (1986).

common stochastic shock (whose precise nature will be specified below), A_t . Second, we also assume there is a positive externality in the production process so that i -th firm's output depends not only on the inputs hired by that firm, but also on the average number of units of capital per worker for the whole economy, $k_t \equiv K_t/N_t$, where $K_t \equiv \int_{[0,1]} K_t^i di$, and $N_t \equiv \int_{[0,1]} N_t^i di$. The consequence is that the economy will display an AK technology in equilibrium, where $Y_t = A_t K_t$, thereby allowing for sustained economic growth which (along the deterministic path) will be constant.

The total factor productivity, A_t , is assumed to be generated by the following process

$$\ln A_t = \hat{A} + \varepsilon_t, \quad [6]$$

where $\varepsilon_t : N(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$ for all t . The normality distribution assumption of ε_t implies that the mean and the variance of A_t are $E(A_{t+1}) = \hat{A}$ and $\text{Var}(A_{t+1}) = \hat{A}^2 \left(e^{\sigma_\varepsilon^2} - 1 \right)$, respectively. The consequence is that changes in σ_ε^2 do affect the variance of A_{t+1} and the certainty-equivalent A_{t+1} , $\left[E(A_{t+1}^{1-\eta}) \right]^{\frac{1}{1-\eta}} = e^{\frac{-\eta}{2}\sigma_\varepsilon^2} \hat{A} \equiv \tilde{A}$, but not its mean¹¹.

The solution to the problem in Eq. [5] is given by the factor price equations

$$r_t = \alpha A_t - \delta, \text{ and } w_t = (1 - \alpha) A_t k_t. \quad [7]$$

In particular, in the non-stochastic steady-state case, the cost of use of capital will be constant, $\alpha \hat{A}$, but the wage rate per unit of labor, $w_t = (1 - \alpha) \hat{A} k_t$, will grow at the same rate as the stock of capital per worker.

1.3. Government

A government sector is introduced in the following way: it taxes capital and labor income at different tax rates in order to finance an exogenous stream of public expenditure, G_t , which, for the sake of analytical convenience, is expressed as a proportion, γ , of aggregate output (i.e. $G_t = \gamma Y_t$). Additionally, we assume, first, that the government balances its budget on a period basis and, second, the government fixes τ_t^k so that, given the realization of the shock A_t , the tax rate τ_t^w is obtained endogenously for a given γ . Thus, for the government budget to be balanced, it must be the case that

$$\gamma Y_t = \tau_t^k (1 + r_t) K_t + \tau_t^w w_t N_t, \quad [8]$$

where the left-hand side stands for government expenditure and the right-hand side represents total tax revenues. Taking into account aggregate production technology in Eq. [5] and factor prices in Eq. [7] yields the following expression for the labor income tax rate

(11) This explains why we assume that the mean of ε_t is $-\sigma_\varepsilon^2/2$ rather than 0. Benabou (2002) makes a similar assumption. Additionally, autocorrelation is assumed away from Eq. [6]. Olovsson (2010) makes the same assumption: although aggregate productivity shocks are highly autocorrelated at annual and quarterly frequencies, evidence of positive serial correlation when 1 model period represents 30 years of calendar time is arguable. See also Heathcote *et al.* (2009).

$$\tau_t^w = \frac{\gamma}{1-\alpha} - \frac{\tau_t^k}{1-\alpha} \left(\frac{\alpha A_t + 1 - \delta}{A_t} \right). \quad [9]$$

Note that, assuming that negative tax rates are not allowed, Eq. [9] imposes upper bounds on τ_t^k and τ_t^w , $\gamma A_t / (\alpha A_t + 1 - \delta)$ and $\gamma / (1 - \alpha)$, respectively.

1.4. Labor market equilibrium

Equilibrium in the labor market is trivially obtained. Using J_t to denote the number of young individuals at t , aggregate labor demand and aggregate labor supply are N_t and $[(1 + n + \theta) / (1 + n)]J_t$, respectively, so that

$$N_t = \left(\frac{1 + n + \theta}{1 + n} \right) J_t, \quad [10]$$

which depends on the labor supply by the adult/old individuals, θ , and the age distribution of the population characterized by n .

1.5. Goods market equilibrium

As is standard in 2-period OLG models with no financial assets at birth, equilibrium in the goods market requires that young individuals' savings be equal to next period's aggregate stock of capital. Therefore, given the equilibrium condition in the labor market, equilibrium in the goods market can be written as

$$s_t = (1 + n + \theta)k_{t+1}. \quad [11]$$

1.6. Growth

Defining the growth rate of capital per unit of labor as $g_t \equiv (k_{t+1} - k_t) / k_t$, it can be shown from Eqs. [2], [3], [4], [7], [9] and [11], and defining $\xi_t \equiv 1 - \tau_t^w$, that

$$1 + g_t = \frac{\xi_t (1 - \alpha) A_t}{1 + n + \theta + \beta^{-\sigma} \left\{ \left[E(\Gamma_{t+1}^{1-\eta}) \right]^{\frac{1}{1-\eta}} \right\}^{1-\sigma} \left\{ E\left[(1 + r_{t+1}^n) \Gamma_{t+1}^{-\eta} \right] \right\}^{-\sigma}}, \quad [12]$$

where

$$\Gamma_{t+1} \equiv (1 + n + \theta)(1 + r_{t+1}^n) + \theta \xi_{t+1} (1 - \alpha) A_{t+1}, \quad [13]$$

and

$$\xi_t \equiv \frac{1 - \alpha - \gamma}{1 - \alpha} + \frac{\tau_t^k}{1 - \alpha} \left(\frac{\alpha A_t + 1 - \delta}{A_t} \right), \quad [14]$$

where $1 + r_{t+1}^n = (1 - r_{t+1}^n) (\alpha A_{t+1} + 1 - \delta)$. Some remarks concerning Eq. [12], which gives us a closed-form solution for g_t follow. First, g_t is known at time t . Second, assuming that the government sets τ_t^k and τ_t^w constant and that $A_t = \hat{A}$ for all t , the right-hand side of Eq. [12] does not depend on t . That is to say, the corresponding growth rate, g_{NS} , is always constant, a key feature of AK growth models.

Third, i) The (gross) rate of growth of y_t is given by $y_{t+1}/y_t = (A_{t+1}/A_t) \times (1 + g_t)$.
 ii) From Eqs. [2], [7] and [11], one has that $c_{1,t}/k_t = (1 - \tau_t^y) (1 - \alpha) A_t - (1 + n + \theta) (1 + g_t)$. And iii) from Eqs. [3], [7] and [11] it follows that $c_{2,t}/k_t = [(1 - \tau_t^y) (\alpha A_t + 1 - \delta)] A_t$. Therefore, if $A_t = \hat{A}$ for all t , then y_t , $c_{1,t}$ and k_t would grow at the same rate, g_{NS} .

Fourth, g_t is a non-linear function of A_{t+1} , so that one should expect that, in general, the non-stochastic steady-state growth rate will differ from the mean of the observed stochastic g_t 's.

Fifth, g_t is i) a linear function of the first period net-of-tax labor income (see $\xi_t (1 - \alpha) A_t$ in the numerator of Eq. [12]), but ii) a non-linear function of the expected net-of-tax interest factor and wage rate (per unit of capital per worker) at $t+1$, $E[(1 - \tau_{t+1}^k) (\alpha A_{t+1} + 1 - \delta)]$ and $E[(1 - \tau_{t+1}^y) (1 - \alpha) A_{t+1}]$, respectively. Therefore, except for the particular case of $\delta = 1$, there is no analytical solution to the stochastic growth rate and its response to aggregate shocks and government policies.

2. EQUILIBRIUM GROWTH RATE

In general, as suggested above, there is no analytical solution to the stochastic growth rate in Eq. [12]. Nevertheless, assuming complete depreciation of physical capital (i.e. $\delta = 1$, a frequent assumption when closed-form solutions are sought), it turns out that g_t can be solved as a reduced form function of the parameters in the model. Needless to say, the log-normality of the aggregate shock, A_t , plays a key role in obtaining that solution. Thus, from Eq. [7] and Eqs. [12-14], it can be shown that the stochastic equilibrium growth rate of per worker stock of capital, k_t , is given by

$$g_t = \frac{\xi_t (1 - \alpha) A_t}{(1 + n + \theta) \left\{ 1 + \Phi_{t+1} [\beta (1 - \tau_{t+1}^k) \alpha]^\sigma \tilde{A}^{1 - \sigma} \right\} - 1}, \quad [15]$$

where $\xi_t \equiv \frac{1 - \gamma - \alpha (1 - \tau_t^k)}{1 - \alpha}$, $\Phi_{t+1} \equiv (1 - \tau_{t+1}^k) \alpha + \theta \xi_{t+1} (1 - \alpha) / (1 + n + \theta)$, $\tilde{A} = e^{\frac{-\eta}{2} \sigma_\epsilon^2}$

\hat{A} is the certainty-equivalent A_{t+1} ¹².

Alternatively, following the discussion in Weil (1990), the young individual's savings, s_t , can be rewritten in terms of the propensity to save out of first-period (net) labor income. Indeed, from the equilibrium condition in Eq. [11] and the definition of g_t , one has that $s_t = (1 + n + \theta) (1 + g_t) k_t$. Therefore, given the expression for in Eq. [15], it can be shown after some mild algebra that young individuals' savings can be written as

$$s_t = \Psi^S w_t^n, \quad [16]$$

where

(12) The proof is in the Appendix.

$$\Psi^S \equiv \frac{1}{1 + \beta^{-\sigma} \left\{ \left(\tilde{R}_{t+1}^n \right)^{1-\sigma} + \theta \Omega \xi_{t+1} (1-\alpha) \left(\tilde{R}_{t+1}^n \right)^{-\sigma} \tilde{A} \right\}} \quad [17]$$

stands for the propensity to save out of first-period net labor income, $w_t^n = \xi_t (1 - \alpha) A_t k_t$, $\tilde{R}_{t+1}^n \equiv E \left[(1 + r_{t+1}^n)^{1-\eta} \right]^{\frac{1}{1-\eta}} = (1 - \tau_{t+1}^k) \alpha \tilde{A}$ denotes the certainty-equivalent net interest factor and, by definition, $\Omega^{-1} \equiv 1 + n + \theta$. As a result, we have that

$$g_t = \Omega \Psi^S \hat{w}_t^n - 1, \quad [18]$$

where $\hat{w}_t^n \equiv w_t^n/k_t = (1 - \tau_t^n) (1 - \alpha) A_t$, i.e. young individuals' disposable labor income (normalized by the stock of capital per unit of labor in the economy). In this way, the growth rate can be explained in terms of i) first-period labor income, and ii) the corresponding savings propensity which depends on both the return of savings and second-period labor income. This will prove useful when we later discuss the effects of changes in τ^k on g_t .

Simple inspection of Eq. [15] shows how increased uncertainty and risk aversion affect young individuals' savings and growth in our model, and the roles played by risk aversion and willingness to intertemporally substitute consumption. The following Proposition gives us the result: all that matters, as in Weil (1990), Mauro (1995) and Smith (1996), is the attitude towards intertemporal substitution of consumption.

Proposition 1. If $\delta = 1$, then i) a mean preserving increment of second period income risk (i.e. an increment in σ_ε^2), and/or ii) a higher risk aversion coefficient increase growth if and only if $IES < 1$.

In other words, the only determinant of the sign of the effect of increased uncertainty on savings is the attitude toward intertemporal substitution, while attitude toward risk affects only the magnitude of the effect. Furthermore, this result does not depend on whether the source of the increased uncertainty is capital or labor income. This is so because uncertainty affects both types of income through the same variable, \tilde{A} . This is most easily understood for the particular case of no second-period labor income, i.e. $\theta = 0$. Therefore, from Eqs. [17] and [18] we obtain

$$g_t = \frac{\Omega \hat{w}_t^n}{1 + \beta^{-\sigma} \left(\tilde{R}_{t+1}^n \right)^{1-\sigma}} - 1. \quad [19]$$

Therefore, σ_ε^2 and η affect the size of the certainty-equivalent interest factor $\tilde{R}_{t+1}^n = (1 - \tau_{t+1}^k) \alpha \tilde{A} = (1 - \tau_{t+1}^k) \alpha e^{\frac{-\eta \sigma_\varepsilon^2}{2}} \hat{A}$, but the sign of individuals' and the economy's responses to changes in \tilde{A} (and, consequently changes in \tilde{R}_{t+1}^n) will depend only on σ^{13} .

(13) More precisely, σ determines the sign of the elasticity of savings with respect to the net-of-tax interest rate, while η affects the magnitude of that elasticity. Therefore, changes in the net-of-tax interest rate will entail an income and a substitution effect on savings; and the income effect is higher than the substitution effect if and only if $\sigma < 1$, i.e. if and only if the interest elasticity of savings is negative.

As for the deterministic case, replacing A_t and \tilde{A} in Eq. [15] with \hat{A} yields the following closed-form solution for the growth rate, g_{NS} , as

$$g_{NS} = \frac{\xi(1-\alpha)\hat{A}}{(1+n+\theta)\left\{1 + \Phi[\beta(1-\tau^k)\alpha]^\sigma \hat{A}^{1-\sigma}\right\}^{-1}}, \quad [20]$$

where $\Phi \equiv (1-\tau^k)\alpha + \theta\xi(1-\alpha)/(1+n+\theta)$, and $\xi \equiv \frac{1-\gamma-\alpha(1-\tau^k)}{1-\alpha}$.¹⁴

In the light of Eqs. [15] and [20], an issue that arises is whether the (expected) stochastic growth rate is higher than the deterministic growth rate or not. The answer is clear: from Proposition 1 we know that uncertainty leads to higher savings if and only if IES is less than 1. Therefore, that must be the necessary and sufficient condition for higher risk levels to give rise to higher (expected) growth rates too. The following Proposition, which extends the result in Mauro (1995) to an economy with a government sector and income taxation, gives the result.

Proposition 2. Assume i) $\delta = 1$, ii) $\tau_t^k = \tau^k$ and iii) $\tau_t^w = \tau^w$. Then $E[g_t] > g_{NS} \Leftrightarrow \sigma < 1$.

Proof. We only have to take into account that, as long as $\eta > 0$ and $\sigma_\varepsilon^2 > 0$, $\tilde{A} \equiv e^{\frac{-\eta}{2}\sigma_\varepsilon^2} \hat{A} < \hat{A}$ so that $\tilde{A}^{\sigma-1} > \hat{A}^{\sigma-1}$ if and only if $\sigma < 1$.

3. CAPITAL INCOME TAXATION AND GROWTH

Here we analyze the conditions on the IES which guarantee a positive monotonic relationship between the capital income tax rate and the growth rate in equilibrium.

The response of g_t on (permanent) changes in τ_t^k will obviously depend on how government tax policy is implemented as three alternatives can be considered following, say, an increment in τ_t^k . First, τ_t^w could be (downwards) adjusted [so that γ remains constant, as in Uhlig and Yanagawa (1996)]. Second, γ could be (upwards) revised for a constant τ^w . And, third, the increment in the capital income tax bill could be lump-sum rebated to old individuals. Here we will only follow the first approach, leaving a discussion on how the results would change for the end of the section.

Assuming an increase in τ_t^k , we decompose the total effect on g_t as the sum of three partial effects. Effect 1: an increment in w_t^n (i.e. an increase in the current disposable income). Effect 2: a drop in \tilde{R}_{t+1}^n (thereby implying, in turn, a positive income effect and a negative substitution effect on the young savings). And Effect 3: a rise in the discounted second-period labor income, $\theta w_{t+1}^n / \tilde{R}_{t+1}^n$. The following

(14) Similarly, Ec. [18] would be replaced with $g_{NS} = \Omega\Psi^{NS} \hat{w}^n - 1$, where now $\hat{w}^n \equiv w_t^n/k_t = (1-\tau^w)(1-\alpha)\hat{A} = \xi(1-\alpha)\hat{A}$ and Ψ^{NS} is defined as Ψ^S except that i) $\tilde{R}_{t+1}^n = (1-\tau_{t+1}^k)\alpha\tilde{A}$ is replaced with $R^n = (1-\tau^k)\alpha\hat{A}$, and ii) \tilde{A} with \hat{A} .

Proposition gives us the expressions (and the signs) for those three effects (I_1 , I_2 and I_3 , respectively) for the stochastic economy under the assumption of full capital depreciation.

Proposition 3. Stochastic economy. Assume i) $\delta = 1$, ii) $\tau_t^k = \tau^k$ and iii) $\tau_t^w = \tau^w$. The total effect of a change in τ^k on g_t can be rewritten as $\frac{\partial g_t}{\partial \tau^k} = I_1^S + I_2^S + I_3^S$, where

$$I_1^S \equiv \Omega \Psi^S \alpha A_t > 0, \quad [21]$$

$$I_2^S \equiv \frac{\Omega(1-\tau^w)(1-\alpha)[\beta(1-\tau^k)]^{-\sigma}[\alpha\tilde{A}]^{1-\sigma}(1-\sigma)A_t}{\tilde{D}^2} > 0 \Leftrightarrow \sigma < 1, \quad [22]$$

where $\tilde{D} \equiv 1 + \beta^{-\sigma} \tilde{A}^{1-\sigma} \{[(1-\tau^k)\alpha]^{1-\sigma} + \Omega\theta\xi(1-\alpha)[(1-\tau^k)\alpha]^{-\sigma}\}$, $\Psi^S = \tilde{D}^{-1}$ and $I_3^S \equiv \frac{-\Omega^2\theta\xi(1-\alpha)[\beta(1-\tau^k)]^{-\sigma}[\alpha\tilde{A}]^{1-\sigma}A_t \left[\frac{\xi(1-\alpha)\sigma}{(1-\tau^k)\alpha} + 1 \right]}{\tilde{D}^2} < 0$. [23]

The same decomposition can be obtained for the deterministic economy by substituting \hat{A} for \tilde{A} in Eqs. [21-23], so that the total effect of a change in τ^k on g_{NS} can be rewritten as $\frac{\partial g_{NS}}{\partial \tau^k} = I_1^{NS} + I_2^{NS} + I_3^{NS}$, where

$$I_1^{NS} \equiv \Omega \Psi^{NS} \alpha \hat{A} > 0, \quad [24]$$

$$I_2^{NS} \equiv \frac{\Omega(1-\tau^w)(1-\alpha)[\beta(1-\tau^k)]^{-\sigma}[\alpha\hat{A}]^{1-\sigma}(1-\sigma)\hat{A}}{\hat{D}^2} > 0 \Leftrightarrow \sigma < 1, \quad [25]$$

where $\hat{D} \equiv 1 + \beta^{-\sigma} \hat{A}^{1-\sigma} \{[(1-\tau^k)\alpha]^{1-\sigma} + \Omega\theta\xi(1-\alpha)[(1-\tau^k)\alpha]^{-\sigma}\}$, $\Psi^{NS} = \hat{D}^{-1}$ and

$$I_3^{NS} \equiv \frac{-\Omega^2\theta\xi(1-\alpha)[\beta(1-\tau^k)]^{-\sigma}[\alpha\hat{A}]^{1-\sigma}\hat{A} \left[\frac{\xi(1-\alpha)\sigma}{(1-\tau^k)\alpha} + 1 \right]}{\hat{D}^2} < 0. \quad [26]$$

The positive sign of I_1^S and I_1^{NS} and the negative sign of I_3^S and I_3^{NS} are trivially rationalized: any increment in τ^k implies a decrease in τ^w and, consequently, increments in both first-period disposable income and discounted second-period net-of-tax labor income. Consequently, savings will increase (resp. fall) according to Effect 1 (resp. Effect 3). The sign of I_2^S and I_2^{NS} , however, depends on the IES, as expected. If consumers dislike (resp. like) to substitute consumption intertemporally, the elasticity of savings with respect to the certainty-equivalent interest factor will be negative (resp. positive). Consequently, as the τ^k increases and falls, savings will increase (resp. decrease) if $IES < 1$ (resp. $IES > 1$), aggregate \tilde{R}^n uncertainty and risk aversion only affecting the magnitude of I_2^S . In sum, if IES

< 1 , Effects 1 and 2 will imply an increase in savings, while Effect 3 will imply a reduction. If $IES > 1$, only Effect 1 implies an increase in savings.

Having discussed the signs of these three partial effects, the following Proposition gives us the conditions under which each of those effects is stronger in a stochastic economy than in a deterministic economy.

Proposition 4. Under the conditions of Proposition 3, and Eqs. [24-26], it is the case that

- i) $E(I_1^S) > I_1^{NS} \Leftrightarrow \sigma < 1$;
- ii) assuming $\eta\sigma_\varepsilon^2 > 0$, $E(I_2^S) > |I_2^{NS}| \Leftrightarrow$

$$\left[e^{-\eta\sigma_\varepsilon^2(1-\sigma)/2} - 1 \right] \left[1 - Ze^{-\eta\sigma_\varepsilon^2(1-\sigma)/4} \right] > 0$$
, where $Z \equiv \beta^{-\sigma} \hat{A}^{1-\sigma}$
 $\left\{ [(1-\tau^k)\alpha]^{1-\sigma} + \Omega\theta\xi(1-\alpha)[(1-\tau^k)\alpha]^{-\sigma} \right\}$; and
- iii) $E(I_3^S) > |I_3^{NS}| \Leftrightarrow E(I_2^S) > |I_2^{NS}|$.

Proof. See Appendix

Proposition 4 needs some further explanation.

Part i) sets the necessary and sufficient condition on the IES under which the expected first effect of a change in τ^k will be stronger in a stochastic than in a deterministic economy. Note that, upon substitution of the certainty-equivalent net-of-tax interest rate, $\bar{R}_{t+1}^n = (1 - \tau_{t+1}^k)\alpha\hat{A}$, in Eq. [17], the propensity to save out of current first-period labor income in the stochastic case, Ψ^S , is higher than that in the deterministic economy, Ψ^{NS} , if and only if the $IES < 1$. If $IES = 1$, the stochastic and the deterministic growth rates coincide [See Proposition 2]. And, if $IES > 1$, “the substitution effect depresses the marginal propensity to save as soon as agents are risk averse” [Weil (1990), page 38]. Consequently, if households dislike to substitute consumption intertemporally, for each additional unit of current disposable income received, households save more under uncertainty ($\Psi^S > \Psi^{NS}$). This result is consistent with the fact that growth is higher in a stochastic framework than in the non-stochastic case [since the precautionary motive is stronger and households save more] if, and only if, $IES < 1$.

Part ii) gives the necessary and sufficient condition for the second effect to be stronger in the stochastic economy than in the deterministic economy. Note, first, that it is straightforward to show that if $\sigma_\varepsilon^2 = 0$ and/or $\eta = 0$, then $E(I_2^S) = |I_2^{NS}|$, as expected. And, second, as discussed concerning Effect 2, if $\sigma = 1$, the expected Effect 2 in the stochastic economy would be identical to the Effect 2 in the deterministic economy¹⁵.

Finally, Part iii) states that the necessary and sufficient condition under which the third effect is stronger in the stochastic setup than in the deterministic

(15) Numerical simulations, although not shown in the paper, indicate that the expected Effect 2 in stochastic economies exceeds that in deterministic economies if and only if $\sigma < 1$.

one coincides with that for the second effect. Simple inspection of Eqs. [22-26] shows why this must be the case. One should bear in mind, however, that Effects 2 and 3 will show opposite signs if and only if $\sigma < 1$.

3.1. Labor income only in the first-period

In order to find analytically, first, the conditions under which a positive monotonic relationship between τ^k and g_{NS} arises in this economy and, second, the conditions under which the effect of changes in τ^k might exhibit a different sign depending on whether the economy is deterministic or stochastic, we further assume the particular case in which old-age individuals earn no labor income, i.e. $\theta = 0$, so that Effect 3 is equal to 0. In the next subsection, we abandon this assumption and analyze the case of positive labor supply in the second period.

We start by obtaining a sufficient (albeit not necessary) condition for increments in the capital income tax rate to induce a higher steady-state growth rate. If $IES < 1$, the savings elasticity with respect to the after-tax interest rate is negative, so that both I_2^S and I_2^{NS} are positive. Therefore, increments in the capital income tax rate will imply a higher steady-state growth rate both in the deterministic and in the stochastic economies.

Proposition 5. Assume $\delta = 1$ and $\theta = 0$. If $\sigma \leq 1$, then $\frac{\partial g_{NS}}{\partial \tau^k} > 0$ and $\frac{\partial g_t}{\partial \tau^k} > 0$.

This is nothing more the main result in Uhlig and Yanagawa (1996) which is here extended to the stochastic economy case¹⁶. Therefore, if $IES < 1$, the growth-maximizing τ^k for a deterministic economy is, as in Caballé (1998)¹⁷. Still, increments in τ^k may induce a higher steady-state growth rate even though $IES > 1$ (so that Effect 2 becomes negative). This will be the case if the decrease in savings, driven by the positive elasticity of savings with respect to the net-of-tax interest rate, is more than offset by the increment in savings motivated by the increase in first-period net-of-tax labor income. The following Proposition gives the result where a necessary and sufficient condition on the structural parameters is obtained.

Proposition 6. Assume $\delta = 1$ and $\theta = 0$. It is the case that

$$\frac{\partial g_t}{\partial \tau^k} = I_1^S + I_2^S > 0 \Leftrightarrow \sigma < \sigma_*^S, \text{ where}$$

$$1 < \sigma_*^S \equiv 1 + \frac{[\beta\alpha(1-\tau^k)]^\sigma \tilde{A}^{\sigma-1} + \alpha(1-\tau^k)}{1-\gamma-\alpha(1-\tau^k)}. \quad [27]$$

Some remarks follow. First, σ can be higher than 1, but not too high (as the negative sign of Effect 2 would prevail upon the positive sign of Effect 1). Second, σ_*^S is decreasing in τ^k , so that the condition in Proposition 6 is more likely to hold for lower values of the capital income tax rate: in fact, one has that $\lim_{\tau^k \rightarrow 1} \sigma_*^S$

(16) See Proposition 1 in Uhlig and Yanagawa (1996), p. 1530.

(17) See Proposition 2 in Caballé (1998), p. 100.

= 1¹⁸. And, third, σ_*^S is decreasing in $\eta\sigma_\varepsilon^2$: for higher levels of risk aversion and aggregate uncertainty it is less likely that increments in capital income taxation induce higher equilibrium growth rates. The result for stochastic economies is trivially extended to deterministic economies by simply replacing \hat{A} with \bar{A} in Eq. [27]. In other words, under the same assumptions applied to δ and θ , one has that

$$\frac{\partial g_{NS}}{\partial \tau^k} = I_1^{NS} + I_2^{NS} > 0 \Leftrightarrow \sigma < \sigma_*^{NS}, \text{ where}$$

$$1 < \sigma_*^{NS} \equiv 1 + \frac{[\beta\alpha(1-\tau^k)]^\sigma \hat{A}^{\sigma-1} + \alpha(1-\tau^k)}{1-\gamma-\alpha(1-\tau^k)}. \quad [28]$$

Therefore, comparing the stochastic and the deterministic economies, one has that, for any given capital income tax rate, $\sigma_*^{NS} > \sigma_*^S$ if and only if $\sigma < 1$. In other words, under the assumptions in Proposition 6, one obtains that it is more likely that an increment in τ^k leads to a higher equilibrium growth rate in the deterministic economy than in the stochastic economy if and only if the IES is greater than 1. This relationship between σ_*^S and σ_*^{NS} will prove useful a few lines below when discussing how aggregate uncertainty and/or risk aversion may reverse the sign of the effect of capital income taxation on growth when compared to that obtained in a deterministic economy.

At this point, a natural question arises. Can the reaction of the growth rate be of opposite signs in stochastic and in deterministic economies, i.e. do aggregate uncertainty and risk aversion play a role as in Smith (1996)? Four cases may be distinguished depending on the size of the IES:

i) If $IES \leq 1$, we have already seen that increments in capital income taxation imply higher steady-state growth rates, both in deterministic and stochastic economies: Effect 1 is (always) positive, and Effect 2 is positive or zero. Recall that the sign of the two effects is the same whether the economy is stochastic or deterministic (See Proposition 3 and Eqs. [24-26]). Therefore, in this case, uncertainty does not play any role in the sign, and only affects the magnitude of the effect.

ii) If $IES > 1$, we know that Effect 2 is negative. We also know that “the substitution effect depresses the marginal propensity to save as soon as agents are risk averse” [see Weil (1990), p. 38]. Therefore, the increment in g_t , driven by the increase in first-period net-of-tax labor income, is lower in the stochastic economy. Thus, $IES > 1$ is a necessary (but not sufficient) condition to obtain a sign reversal, as three cases may arise. In what follows, the reader is requested to bear in mind, first, the results in Proposition 3 and Eqs. [24-26]; second, Proposition 6 and Eq. [28]; and, third, that σ_*^S and σ_*^{NS} there obtained depend, in turn, on τ^k , so that sign reversals will be obtained for particular ranges of the capital income tax rate.

(18) The condition in Proposition 6 can be rewritten in terms of the capital income tax rate. Thus, it can be shown that, under the same assumptions, an increase in τ^k leads to a higher equilibrium growth rate in the stochastic economy if and only if $\tau^k < 1 + (\sigma\alpha)^{-1} \{[\beta\alpha(1-\tau^k)^\sigma \bar{A}^{\sigma-1} + (1-\sigma)(1-\gamma)]\}$, i.e. τ^k is low enough, so that g_t and τ^k follow an inverted-U pattern.

ii.a) If $1 < IES \leq \sigma_*^S < \sigma_*^{NS}$, increments in τ^k will imply a non-negative effect on growth in the stochastic economy and a positive one in the deterministic economy. Therefore, aggregate uncertainty and/or risk aversion do not give rise to a sign reversal, only affecting the size of the effect on growth.

ii.b) If $1 < \sigma_*^S < IES < \sigma_*^{NS}$, increments in τ^k will lead to a negative effect on growth in the stochastic economy, but a positive one in the deterministic economy. That is to say, it may be the case that for a high enough relative risk aversion and/or aggregate uncertainty, increments in capital income taxation might give rise to opposite sign effects.

ii.c) Finally, if $1 < \sigma_*^S < \sigma_*^{NS} \leq \sigma$, increases in τ^k will imply a negative effect on growth in the stochastic economy and a non-positive effect on growth in the deterministic economy. Therefore, as in cases i) and ii.a), aggregate uncertainty and/or risk aversion will have no effect to the sign of the reaction of the growth rate to changes in capital income taxation, but only to its magnitude.

The following Proposition summarizes the last results.

Proposition 7. Assume $\delta = 1$ and $\theta = 0$.

i) If $\sigma \leq 1 \Rightarrow \partial g_t / \partial \tau^k > 0$ and $\partial g_{NS} / \partial \tau^k > 0$.

ii) Assume $\sigma > 1$: ii.a) if $\sigma \leq \sigma_*^S < \sigma_*^{NS} \Rightarrow \partial g_t / \partial \tau^k \geq 0$ and $\partial g_{NS} / \partial \tau^k > 0$.

ii.b) if $\sigma_*^S < \sigma < \sigma_*^{NS} \Rightarrow \partial g_t / \partial \tau^k < 0$ but $\partial g_{NS} / \partial \tau^k > 0$.

ii.c) if $\sigma_*^S < \sigma_*^{NS} \leq \sigma \Rightarrow \partial g_t / \partial \tau^k < 0$ and $\partial g_{NS} / \partial \tau^k \leq 0$.

3.2. Labor income in the second period

If households earn labor income in their second period, so that $\theta > 0$, then the (negatively signed) Effect 3 comes into the picture. If θ is close to 0, the magnitude of Effect 3 will be low (both in deterministic and stochastic economies). If $IES < 1$, Effect 2 will be positive, and one would expect that increments in the capital income tax rate will lead to higher steady-state growth rate. For values of θ close to 1, however, increments in τ^k might induce reductions in the equilibrium growth rate as, in fact, numerical simulations conducted under reasonable parameter values for the rest of the structural parameters will show in Section 4.

We proceed now to obtain a sufficient condition for increments in the capital income tax rate to induce a higher stochastic steady-state growth rate: as expected, a low enough IES guarantees that the income effect of the fall in the net interest factor more than offsets the substitution effect of the fall in the net interest factor and the negative effect coming from the increase in the discounted second-period labor net income¹⁹.

(19) Alternatively, the condition on σ in Proposition 8 can also be rewritten in terms of an upper bound for τ^k as follows. Assume i) $\delta = 1$, and ii) $\sigma < \frac{\alpha(1+n)}{\alpha(1+n)+\theta(1-\gamma)} < 1$. If $\tau^k < \tau_*^k$ where

$\tau_*^k \equiv 1 - \frac{\sigma\theta(1-\gamma)}{(1-\sigma)\alpha(1+n)} \in (0,1)$ then $\frac{\partial g_t}{\partial \tau^k} > 0$. It can be shown that, had we considered the alternative

approach of keeping τ^k constant and γ variable, a low value of σ would be sufficient and *necessary* and, similarly, a growth maximizing τ^k could be obtained. Of course, Effect 1 would vanish.

Proposition 8. Assume $\delta = 1$. If $\sigma \leq \sigma_*(\theta)$, where

$$\sigma_*(\theta) \equiv \frac{\alpha(1-\tau^k)(1+n)}{\alpha(1-\tau^k)(1+n)+\theta(1-\gamma)} < 1, \quad [29]$$

then $\frac{\partial g_t}{\partial \tau^k} > 0$.

Proof. See Appendix.

Regarding Proposition 8, two remarks are in order. First, σ less than $\sigma_*(\theta)$ is not necessary by the same reasoning used for $\theta = 0$. Second, the sufficiency nature of the condition implies that the main determinant of the sign of $\frac{\partial g_t}{\partial \tau^k}$ is the IES, while the aggregate uncertainty and individuals' risk aversion parameters (σ_ε^2 and η) do not play any role in this bound $\sigma_*(\theta)$. Note, finally, that the higher θ , the lower the right-hand side in Eq. [29], thus making it less likely that increments in τ^k give rise to higher growth rates: if θ is high enough, the increment in second-period disposable labor income might end up reducing first-period savings.

Similarly, as we proceeded for the $\theta = 0$ case, we can, first, characterize a necessary and sufficient condition for the model parameters such that increments in τ^k in give rise to higher equilibrium growth rates in stochastic economies. Second, repeat the exercise for the deterministic economy. And, third, compare the two economies.

The following Proposition provides us with the first result.

Proposition 9. Assume $\delta = 1$. It is the case that

$$\frac{\partial g_t}{\partial \tau^k} = I_1^S + I_2^S + I_3^S > 0 \Leftrightarrow \sigma < \sigma_*^S(\theta), \text{ where}$$

$$\sigma_*^S(\theta) \equiv \frac{\alpha(1-\tau^k)(1+n+\theta) \{ [\beta\alpha(1-\tau^k)]^\sigma \tilde{A}^{\sigma-1} + 1 - \gamma \}}{[1-\gamma-\alpha(1-\tau^k)] \times [\alpha(1-\tau^k)(1+n)+\theta(1-\gamma)]}.$$

Some remarks follow. First, it is straightforward to check that if $\theta = 0$, this condition coincides with that stated in Proposition 6 as expected. As θ increases, the negative effect deriving from the increment in the discounted second-period net labor income (i.e. Effect 3) will be stronger and, eventually, will offset the positive effect of the first-period labor income and of the income effect of the fall in the interest factor. Consequently, in such a case, a lower IES will be needed in order for the income effect of the fall in the interest factor to be large enough.

Second, it is possible to obtain a similar condition for the deterministic economy too: simply replace \tilde{A} with \hat{A} in $\sigma_*^S(\theta)$ in Proposition 9. Therefore, once again assuming that $\delta = 1$, one has that $\frac{\partial g}{\partial \tau^k} = I_1^{NS} + I_2^{NS} + I_3^{NS} > 0 \Leftrightarrow \sigma < \sigma_*^{NS}(\theta)$, where

$$\sigma_*^{NS}(\theta) \equiv \frac{\alpha(1-\tau^k)(1+n+\theta)\{\beta\alpha(1-\tau^k)\}^\sigma \hat{A}^{\sigma-1} + 1 - \gamma}{[1-\gamma-\alpha(1-\tau^k)] \times [\alpha(1-\tau^k)(1+n)+\theta(1-\gamma)]}. \quad [30]$$

Third, in contrast to the $\theta = 0$ case, there is no clear cut conclusion about how $\sigma_*^S(\theta)$ compares to 1. Given that $\partial\sigma_*^S(\theta)/\partial\theta < 0$ and that $\sigma_*^S(\theta = 0) < 1$, a negative relationship between τ^k and g_t might arise even for $IES < 1$ as numerical simulations will show in Section 4.

Fourth, as occurred when $\theta = 0$, given that $\tilde{A} < \hat{A}$ for $\eta\sigma_\varepsilon^2 > 0$, by comparing the numerators of expressions for $\sigma_*^S(\theta)$ and $\sigma_*^{NS}(\theta)$ in Proposition 9 and Eq. [30], respectively, we obtain that $\sigma_*^{NS}(\theta) > \sigma_*^S(\theta)$ if and only if $\sigma > 1$.

Fifth, and recovering the main issue dealt with in this paper: what is the role played by aggregate uncertainty when considering tax policy? Or, in other words, is it possible to obtain different sign results upon tax policy upon the equilibrium growth rate in stochastic and in deterministic economies? The following Proposition gives us the result: sufficient joint conditions on the σ are obtained under which a reversal in the sign of tax policy takes place when introducing aggregate uncertainty in a deterministic economy.

Proposition 10. Assume $\delta = 1$ and $\theta > 0$:

i) Assume: $\sigma > 1$: i.a) if $\sigma \leq \sigma_*^{NS}(\theta) < \sigma_*^S(\theta)$, then $g_t/\partial\tau^k \geq 0$ and $\partial g_{NS}/\partial\tau^k > 0$; i.b) if $\sigma_*^{NS}(\theta) < \sigma < \sigma_*^S(\theta)$, then $g_t/\partial\tau^k > 0$ but $\partial g_{NS}/\partial\tau^k < 0$; i.c) if $\sigma_*^{NS}(\theta) < \sigma_*^S(\theta) \leq \sigma$, then $g_t/\partial\tau^k \leq 0$ and $\partial g_{NS}/\partial\tau^k < 0$;

ii) Assume: $\sigma > 1$: ii.a) if $\sigma \leq \sigma_*^S(\theta) < \sigma_*^{NS}(\theta)$, then $\partial g_t/\partial\tau^k < 0$ and $\partial g_{NS}/\partial\tau^k \geq 0$; ii.b) if $\sigma_*^S(\theta) < \sigma < \sigma_*^{NS}(\theta)$, then $\partial g_t/\partial\tau^k < 0$ but $\partial g_{NS}/\partial\tau^k > 0$; ii.c) if $\sigma_*^S(\theta) < \sigma_*^{NS}(\theta) \leq \sigma$, then $\partial g_t/\partial\tau^k \leq 0$ and $\partial g_{NS}/\partial\tau^k < 0$, where $\sigma_*^S(\theta)$ and $\sigma_*^{NS}(\theta)$ have been defined in Proposition 9 and Eq. [30], respectively.

The intuition for the sign reversal result is as follows. If the income effect of the decrease in the after-tax interest rate is high enough ($\sigma < 1$), it may happen (for some range of τ^k) that increments of τ^k decrease the steady-state growth rate in a deterministic economy, but under uncertainty, the equilibrium growth rate rises. In our economy, the range of τ^k for which the reversal of the sign may happen depends on aggregate uncertainty and individuals' risk aversion parameters (σ_ε^2 and η). Conversely, if the substitution effect of the decrease in the after-tax interest rate is high enough ($\sigma > 1$), it may happen that increments of τ^k lead to higher growth rates in a deterministic economy, but lower in stochastic economies. A consequence of Propositions 7 and 10 is that, in order to obtain a sign reversal when $\sigma < 1$, θ must be positive; and this is so because, if $\sigma < 1$, both Effect 1 and Effect 2 are positive. All these results will be conveniently illustrated in the numerical simulation exercises carried out in Section 4.

3.3. Alternative tax policies

So far we have considered a particular way of implementing capital income tax policy, as we have assumed that changes in τ^k have been accompanied by changes in the labor income tax rate in such a way that the government spending to GDP ratio, γ , has been kept constant. A natural question arises: are the results thus obtained robust to alternative implementations of the capital income tax policy? Two

obvious alternatives are considered in turn: i) increments in τ^k are not accompanied by reductions in τ^k , so that γ is accordingly increased; and ii) increments in τ^k are accompanied by increments in lump-sum transfers to individuals in their second period of life. In both cases, we will only present the main results for the sake of space saving, all the details being available from the authors upon request.

• Increments in τ^k are not accompanied by reductions in τ^w . If this were the tax policy implemented, then the following results can be proven.

Proposition 11. Assume i) $\delta = 1$, and ii) $\tau_t^w = \tau^w$ for all t . Then $\left. \frac{\partial g_t}{\partial \tau^k} \right|_{\text{constant } \tau^w} > 0$ if and only if $\sigma < \sigma^{**}$, where

$$\sigma^{**} \equiv \frac{\alpha(1-\tau^k)(1+n+\theta)}{\alpha(1-\tau^k)(1+n+\theta) + \theta(1-\tau^w)(1-\alpha)} < 1. \quad [31]$$

Going back to our discussion on the split of the total effect of capital income taxation, Effect 1 disappears; Effect 2 stays the same, so that its sign will depend on how σ relates to 1; and the size of Effect 3 slightly changes (second-period, net-of-tax labor income does not change, although its present value does), but its sign remains negative. That is why a low enough IES is not only sufficient, but also necessary.

The following Proposition characterizes the growth-maximizing capital income tax rate (and the maximum growth rate) for some given labor income tax rate, provided that $\sigma < 1$.

Proposition 12. Assume i) $\delta = 1$, ii) $\tau_t^w = \tau^w$ for all t and iii) $\sigma < 1$. Then

$\left. \frac{\partial g_t}{\partial \tau^k} \right|_{\text{constant } \tau^w} > 0 \Leftrightarrow \tau^k < \tau^{**}$, where

$$\tau^{**} \equiv 1 - \frac{\theta\sigma(1-\tau^w)(1-\alpha)}{\alpha(1+n+\theta)(1-\sigma)} < 1,$$

and the maximum growth rate is given by

$$g_t^{**} = \frac{(1-\tau^w)(1-\alpha)A_t}{(1+n+\theta) \left\{ 1 + (\beta\sigma)^{-\sigma} \left[\frac{\theta(1-\tau^w)(1-\alpha)\tilde{A}}{(1+n+\theta)(1-\sigma)} \right]^{1-\sigma} \right\}} - 1.$$

Therefore, this time it is possible to characterize an upper bound for the IES, σ^{**} , which is both necessary and sufficient for increments in capital income tax to induce higher equilibrium growth rates. Moreover, aggregate uncertainty and risk are irrelevant for the sign of $\left. \frac{\partial g_t}{\partial \tau^k} \right|_{\text{constant } \tau^w}$ because bound σ^{**} does not depend on η and σ_ε^2 : this implies that capital income tax policy will induce growth effects of the same sign in stochastic and in deterministic economies. In other words: there is no sign reversal of the income tax policy when changes in τ^k are accompanied by constant τ^w and varying γ . Uncertainty, and risk aversion, play a (minor) role, however: the maximum expected equilibrium growth rate in the stochastic

economy will be higher than the maximum equilibrium growth rate in a deterministic economy if and only if $\sigma < 1$, as already pointed out in Proposition 2.

- Increments in τ^k are lump-sum rebated to individuals in their second period. Without loss of generality, here we assume that labor income goes untaxed (i.e. $\tau^v = 0$). If this were the tax policy implemented, then the following results can be proven.

Proposition 13. If $\delta = 1$, and the capital income tax bill is lump sum rebated to individuals in their second period, then the stochastic equilibrium growth rate of per worker stock of capital is given by

$$g_t = \frac{(1-\alpha)A_t}{(1+n+\theta)\left\{1 + \hat{\Phi}_{t+1} \left[\beta(1-\tau_{t+1}^k)\alpha\right]^{-\sigma} \tilde{A}^{1-\sigma}\right\}} - 1,$$

where $\hat{\Phi}_{t+1} \equiv \alpha + \theta(1-\alpha)/(1+n+\theta)$, $\tilde{A} = e^{\frac{-\eta}{2}\sigma^2} \hat{A}$ is the certainty-equivalent A_{t+1} .

The consequence is a negative monotonic relationship between the capital income tax rate and the equilibrium growth rate, so that there is no sign reversal of the effect of changes in τ^k on this occasion either. Therefore, uncertainty and risk aversion only matter for the size of the tax effect. Once again, the result in Proposition 2 extends to this alternative tax policy. In this setup, it can be shown that Effect 1 is 0 by construction; Effect 2 is positive if and only if $\sigma < 1$ as in all previous tax policies considered. Effect 3 (provided that $\theta > 0$) is negative as the increment in τ^k increases the discounted value of second-period, net-of-tax labor income. An additional (negative) effect, that we might call Effect 4, appears this time: the transfer obtained in the second period would reduce first-period savings.

So far our results have been exclusively analytical. In the next section we complete our analysis with a numerical exercise.

4. A NUMERICAL ILLUSTRATION

4.1. Calibration

Here we set the parameter values we use in our numerical exercises in this section²⁰. In choosing the appropriate values, we will try to mimic some stylized facts of a national economy to some extent, namely, the US economy, by far the most frequently considered in numerical exercises like the one we run here. The values are summarized in Tables I and II.

4.1.1. Parameters exogenously determined

- Preferences. We set $\beta = 0.55$. We obtain this value by assuming a yearly subjective discount factor of 0.98, a standard value in the literature, and that 1 period represents years. We set $\eta = 4$, as in Dibooglu and Kenc (2009) and Conesa

(20) The analysis performed in this section should be understood as a mere numerical illustration of the quantitative implications of our model economy, as the two-period OLG model is certainly not an adequate instrument to perform calibration and simulation exercises aimed at matching data.

et al. (2009), although the latter do not distinguish between risk attitude and intertemporal substitution. Estimates of η in Cagetti (2001) range between 3.22 and 8.13, depending on the database used, the individuals' educational attainment and whether or not housing wealth is included among assets. As in Uhlig and Yanagawa (1996), we set $\sigma = 0.5$, which implies "an [IES] centered around the median of the estimates in the literature".

Smith (1996) finds that following an increment in the capital income tax rate the risk aversion coefficient has to be implausibly high ($\eta = 16.5$) to obtain an increase in the growth rate in the stochastic economy, but a decrease in the growth rate in the deterministic economy. We will also consider an extremely high value (namely, $\eta = 48.0$)²¹ in order to make the stochastic economy differ enough from the deterministic one and, thereby, illustrate the possibility of sign reversals in the effect of τ^k upon the growth rate. And with that same purpose, we will also consider alternative values of $\sigma = \{0.75, 1.5, 2.0\}$.

- **Technology.** We assume a standard $\alpha = 0.36$ and complete capital depreciation, which we believe is a reasonable assumption given the time length represented by one period in this economy²².

- **Government.** Finally, we set the government consumption share of GDP, γ , equal to 17%. [See, e.g. Conesa and Krueger (2006)]. As for the tax rates, given our assumption of government budget balance, we set $\tau_w = \tau_k = 0.17$, certainly lower than observed. Alternatively, we might have set τ_w and τ_k equal to the observed marginal tax rates for labor and capital income. Thus, following Klein *et al.* (2005), we might have assumed $\tau_w = 0.226$ and $\tau_k = 0.510$. Following that procedure, however, would imply that (assuming budget balance) government spending would represent (given Eq. [8]) 32.1% of GDP, well above the observed value.

4.1.2. Parameters endogenously determined (calibrated)

- **Demographics.** According to the US Census Bureau, the average annual population growth rate between 2000 and 2008 was 0.009382; assuming that each of the two periods in our model represents years, we set $n = 0.32333$. We set θ equal to 0.4. This results from assuming, as in Conesa *et al.* (2009), that individuals are born at 20, and, according to the U.S. Census Bureau, that the average effective retirement age is 62, so that the fraction of inelastic second-period labor supply is $(62 - 50)/30$. As we will do with η and θ , and with the purpose above explained, we will consider alternative values for $\theta = \{0, 0.05, 0.98\}$.

- **Technology.** We set $\hat{A} = 42.38$ such that we obtain a (30-year) growth rate of 68.3%, the observed value for an annual growth rate of 1.75%, the value assumed in Conesa and Krueger (2006). Concerning the variance of ε_t in Eq. [6], we

(21) See Caldara *et al.* (2012).

(22) Note that $\delta = 1$ is a necessary assumption to obtain the analytical results and to compute the partial effects of changes in τ^k discussed in the previous section, although the total effect can be numerically computed without complete depreciation. We ran the numerical exercise for $\delta < 1$, even though we do not show the details for the sake of space saving. The conclusion is neat: our results are robust to the complete depreciation assumption.

set $\sigma_{\varepsilon}^2 = 0.006336$. In this way, the variance of the (30-year period) per capita growth rate that we obtain turns out to match the observed value for the period 1820-2000, 0.018²³. In order to illustrate our point, we will also consider the deterministic case (i.e. $\sigma_{\varepsilon}^2 = 0.0$).

Table I: PARAMETER VALUES

Preferences:	$\beta = 0.55$	$\eta = 4$	$\sigma = 0.5$	
Demographics	$n = 0.323$	$\theta = 0.4$		
Technology:	$\alpha = 0.36$	$\delta = 0.36$	$\hat{A} = 42.38$	$\sigma_{\varepsilon}^2 = 0.006336$
Government:	$\gamma = 42.38$	$\tau^v = 0.17$	$\tau^k = 0.17$	

Source: $\beta, \eta, \sigma, \alpha, \gamma$ and the τ 's have been exogenously set; the other parameters have been endogenously calibrated by the authors with Matlab[®].

Table II: SIMULATED AND TARGET VALUES

Variable	Simulated	Target
Average Per Capita Growth rate (annual)	1.75%	1.75%
Per Capita Growth rate variance	0.018	0.018
Population Growth rate (annual)	0.938%	0.938%

Source: The target values for the average per capita growth rate and the population growth rate have been obtained from Conesa and Krueger (2006) and the US Census Bureau, respectively. The target value for the per capita growth rate variance has been computed by the authors using data from Statistics on World Population, GDP and Per Capita GDP, 1-2008 AD, by Angus Maddison, available at <http://www.ggdcc.net/maddison>. Simulated equilibrium values have been obtained by the authors with Matlab[®].

4.2. Findings

The conditions under which the uncertainty either reinforces or offsets the response of the growth rate to changes in the capital income tax rate is not so straightforward as in Smith (1996). This is because, in our model economy, changes in τ^k raise the above mentioned three effects on growth, the total effect depending on the signs of the differences between the stochastic and the deterministic case for each of the effects.

We show our results in Table III and in Figure I. In Table III, we show the capital income tax rates at which equilibrium growth is maximized for both the stochastic and the deterministic economy, $\hat{\tau}_k^S$ and $\hat{\tau}_k^{NS}$, respectively, and the corres-

(23) See Statistics on World Population, GDP and Per Capita GDP, 1-2008 AD, by Angus Maddison, available at <http://www.ggdcc.net/maddison>.

Table III: GROWTH RATES. STOCHASTIC VS. DETERMINISTIC ECONOMY

σ	θ	$\hat{\tau}_k^S$	$\hat{\tau}_k^{NS}$	$E(g_S^*)$	g_{NS}^*	$\sigma_*^S(\theta)$	$\sigma_*^{NS}(\theta)$	$E[\partial g_S / \partial \tau^k]$	$\partial g_{NS} / \partial \tau^k$
0.50	0.00	0.472*	0.472*	1.511	1.381	1.429	1.419	3.058	2.936
	0.05	0.472*	0.472*	1.435	1.302	1.265	1.257	2.576	2.458
	0.40	0.472*	0.472*	1.143	1.006	0.775	0.771	0.963	0.899
	0.95	0.472*	0.472*	0.977	0.842	0.561	0.560	0.213	0.194
0.75	0.00	0.472*	0.472*	1.240	1.184	1.451	1.446	1.958	1.918
	0.05	0.472*	0.472*	1.167	1.110	1.290	1.285	1.525	1.489
	0.40	0.472*	0.472*	0.896	0.839	0.802	0.799	0.149	0.138
	0.95	0.346	0.344	0.775	0.718	0.750	0.747	0.000	-0.006
1.50	0.00	0.434	0.445	0.687	0.772	1.500	1.514	0.000	0.027
	0.05	0.343	0.353	0.649	0.731	1.500	1.517	0.000	0.028
	0.40	0.135	0.146	0.612	0.690	1.500	1.527	0.000	0.033
	0.95	0.048	0.060	0.632	0.710	1.500	1.534	0.000	0.037
2.00	0.00	0.154	0.182	0.555	0.695	2.000	2.069	0.000	0.075
	0.05	0.115	0.143	0.558	0.696	2.000	2.077	0.000	0.078
	0.40	0.008	0.036	0.597	0.728	2.000	2.108	0.000	0.093
	0.95	0.000*	0.000*	0.627	0.754	1.876	1.997	-0.103	-0.003

Key to Table III: The table shows τ_k^S and τ_k^{NS} , the capital income tax rates at which equilibrium growth is maximized for both the stochastic and the deterministic economy ($\hat{\tau}_k^S$ and $\hat{\tau}_k^{NS}$, respectively), and the corresponding maximum growth rates, $E(g_S^*)$ and g_{NS}^* , for the indicated combinations of σ and θ . Parameters \hat{A} and σ_ε^2 have been recalibrated for each (σ, θ) pair, and η has been set equal to 48. An asterisk (*) in Columns 3 and 4 indicates that both growth rates are maximized at either $\tau_k = \gamma/\alpha$ or $\tau_k = 0$, i.e. the upper and lower bounds for τ_k in our experiment. Columns 7 and 8 show the right-hand sides of the inequality in Proposition 9 and Eq. [30], respectively, both evaluated at $\tau_k = \hat{\tau}_k^S$. Columns 9 and 10 show derivatives of the expected growth rate in the stochastic economy and that of the growth rate in the non-stochastic economy with respect to the capital income tax rate, respectively, both evaluated at $\tau_k = \hat{\tau}_k^S$. Figures have been obtained by the authors using Matlab[®].

pondering maximum growth rates, $E(g_S^*)$ and g_{NS}^* , for combinations of σ and θ . The average total factor productivity and the variance of the perturbation in Eq. [6], \hat{A} and σ_ε^2 , have been recalibrated for each of the combinations. And (in order to force the stochastic economy to differentiate from the deterministic one in an illustrative way), the risk aversion parameter, η , has been set equal to 48, i.e. 12 times higher than the benchmark value. In Figure I, we show the growth rates for three different values of the product $\eta \times \sigma_\varepsilon^2$: i) deterministic case, $\eta \times \sigma_\varepsilon^2 = 0$, ii) the benchmark value, $\eta \times \sigma_\varepsilon^2 = 4 \times 0.006$, and iii) $\eta \times \sigma_\varepsilon^2 = 48 \times 0.006$, and for two alternate values of σ : 0.5 (Figure I.a) and 1.5 (Figure I.b).

Our numerical results can be summarized as follows.

First, as stated in Proposition 2, expected growth exceeds the deterministic growth rate if and only if $IES < 1$. See Figures I.a and I.b, and Table III, Columns 5 and 6 for those cases in which the τ^k rate considered is the same for the stochastic and the deterministic economies: rows 1-7 and 16²⁴.

Second, we can see in Table III that, if $\sigma = 0.5$ or $\sigma = 0.75$ (i.e. $IES < 1$) and, in the latter case, θ is low enough ($\theta \leq 0.4$), then $\hat{\tau}_k^S = \hat{\tau}_k^{NS} = \gamma\alpha$ and $\sigma < \sigma_*^{NS}(\theta) < \sigma_*^S(\theta)$. The growth rate in both the stochastic and the deterministic economy increases as the capital income tax rate increases (both $E[\partial g_t / \partial \tau^k]$ and $\partial g_{NS} / \partial \tau^k$ are positive), and there is no reversal of the sign [see Proposition 10, part i)]. [See rows 1-7 and columns 7-10].

Third, if $\sigma = 0.75$ (so that still $IES < 1$) but θ is close to 1 (e.g. 0.95), then $\gamma\alpha > \hat{\tau}_k^S > \hat{\tau}_k^{NS} > 0$: a sign reversal appears. Whenever θ rises, both $\sigma_*^S(\theta)$ and $\sigma_*^{NS}(\theta)$ fall, so that both $E(g_t)$ and g^{NS} follow an inverted-U pattern relative to τ^k . Note that evaluating $\sigma_*^S(\theta)$ and $\sigma_*^{NS}(\theta)$ at $\tau_k = \hat{\tau}_k^S$ yields $1 > \sigma = \sigma_*^S(\theta) > \sigma_*^{NS}(\theta)$: it must be the case that a (marginal) increment in τ^k will leave the expected stochastic growth rate unaffected, but will reduce the deterministic economy growth rate (i.e. and $E[\partial g_t / \partial \tau^k] = 0$, and $\partial g_{NS} / \partial \tau^k = 0.006$). [See row 8 and columns 3, 4, 7-10].

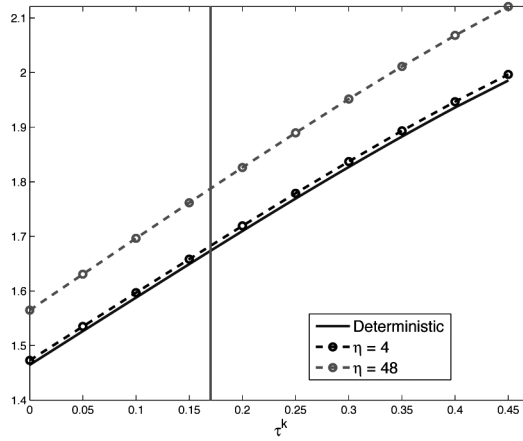
Fourth, if $\sigma = 1.5$ or $\sigma = 2$ (so that $IES > 1$ on this occasion), and in the latter case θ is not too high (less than or equal to 0.4 in our experiment), both $E(g_t)$ and g^{NS} follow an inverted-U pattern relative to τ^k . This time, it turns out that $0 < \hat{\tau}_k^S < \hat{\tau}_k^{NS} < \gamma\alpha$, so that a sign reversal rises again. Thus, evaluating $\sigma_*^S(\theta)$ and $\sigma_*^{NS}(\theta)$ at $\tau_k = \hat{\tau}_k^S$ at yields: therefore an increment in τ^k will leave the expected stochastic growth rate unchanged, but will increase the deterministic economy growth rate (i.e. and $E[\partial g_t / \partial \tau^k] = 0$, and $\partial g_{NS} / \partial \tau^k < 0$). [See rows 8-10 and columns 3, 4, 7-10].

Fifth, if $\sigma = 2$ and θ is high enough (0.95 in our exercise), then we obtain that $\hat{\tau}_k^S = \hat{\tau}_k^{NS} = 0$: the growth rate is maximized at a 0 capital income tax rate, so that no sign reversal can occur in this case as increments in τ_k will always (for all $\tau_k \in 0, \gamma\alpha$) lead to drops in the equilibrium growth rate. The other numerical results obtained for this case are consistent with that: $1 < \sigma_*^S(\theta) < \sigma_*^{NS}(\theta)$, and both $E[\partial g_t / \partial \tau^k]$ and $\partial g_{NS} / \partial \tau^k$ are negative.

Sixth, for reasonable values of the IES (i.e. less than 1), a sign reversal only appears for unplausibly high values of the second-period labor supply, $\theta = 0.95$ [see row 8 and columns 9-10]. For values of the $IES > 1$, however, sign reversals appear for all the values of θ considered [see rows 9-15 and columns 9-10]. The conclusion is a neat one: deterministic OLG economies are a good approximation to the effects of taxes on growth as in Smith (1996).

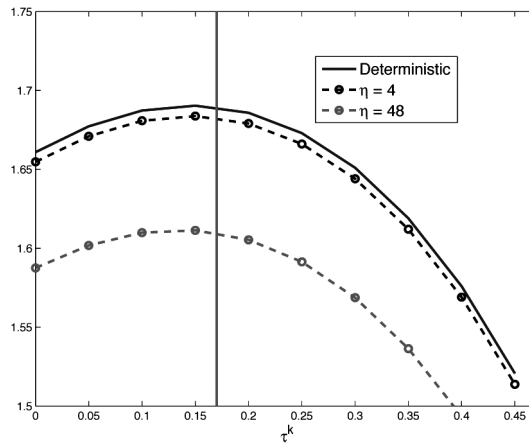
(24) The other rows are not directly comparable as they show expected stochastic growth rates and deterministic growth rates for different capital income tax rates.

Figure 1a: AVERAGE GROWTH. STOCHASTIC VS. DETERMINISTIC.
 $1 + E[G]$. IES = 0.5



Source: Simulations by the authors using Matlab[®].

Figure 1b: AVERAGE GROWTH. STOCHASTIC VS. DETERMINISTIC.
 $1 + E[G]$. IES = 1.5



Source: Simulations by the authors using Matlab[®].

5. CONCLUSIONS

In this paper, we built a two-period, OLG economy with aggregate uncertainty in which individuals' attitudes towards intertemporal substitution and risk aversion are treated separately, and where aggregate technology in equilibrium displays an AK pattern, so that young individuals' savings, with a precautionary component, determine the sustainable output growth rate. Under the assumption that the government levies proportional taxes on capital and labor incomes at different rates and balances its budget at each period, we study the effect of changes in the capital income tax rate on the expected equilibrium growth rate.

Within this setup, our main results follow:

1. We have broken down the total effect of capital income taxation as the sum of three partial effects on i) current net labor income, ii) future net capital income, and iii) future discounted net labor income. We have obtained conditions that, first, guarantee the signs of each of these effects and, second, determine whether these are stronger under uncertainty or perfect foresight.

2. We have obtained necessary and sufficient conditions for increments in the capital income taxation to induce higher equilibrium growth rates in stochastic and in deterministic economies.

3. Assuming that households' second-period labor supply is zero, we have obtained a necessary and sufficient condition, implying that the IES is higher than one, such that increments in the capital income tax rate will lead to higher steady-state growth rates in deterministic economies, but lower rates in stochastic economies. If the IES is less than one, however, increases in the capital income tax will always induce higher equilibrium growth rates, both in stochastic and non-stochastic economies.

4. If individuals' second-period labor supply is positive, we have obtained two necessary and sufficient conditions for increments in the capital income tax rate to have opposite sign effects on the equilibrium growth rate in stochastic and in deterministic economies, depending on how IES compares to one. If IES is less (resp. higher) than one, increments in capital income taxation may generate higher growth rates in stochastic economies, but lower rates in deterministic ones (resp. lower growth rates in stochastic economies, but higher rates in deterministic ones).

5. We have analytically shown that no sign reversal can be obtained under either of the two alternative tax policies discussed: i) set the labor income tax rate constant and let the government tax proceeds change accordingly, or ii) lump-sum rebate the income tax revenues to second-period individuals.

6. We have built a numerical example after calibrating our model economy to mimic some stylized facts of the U.S. economy to illustrate our results. We have shown the critical roles played by both the IES and the second-period labor supply in obtaining a sign reversal. For reasonable values of the IES (i.e. less than 1), a sign reversal only appears for implausibly high values of the second-period labor supply. For values of the IES > 1 , however, reversals of the sign arise for all second-period labor supply values considered. The clear conclusion is, as stated earlier, that deterministic OLG economies are a good approximation to the effects of taxes on growth as in Smith (1996).

APPENDIX

• Proof of the Equilibrium growth rate

Consider Eqs. [12-14] and assume that $\delta = 0$. Thus we obtain that

$$1 + g_t = \frac{\xi_t(1-\alpha)A_t}{1+n+\beta^{-\sigma} \left\{ \left[E(\Gamma_{t+1}^{1-\eta}) \right]^{\frac{1}{1-\eta}} \right\}^{1-\sigma\eta} \left\{ E\left[(1+r_{t+1}^n) \Gamma_{t+1}^{-\eta} \right] \right\}^{-\sigma}}, \quad [32]$$

where

$$\Gamma_{t+1} \equiv (1+n+\theta)(1+r_{t+1}^n) + \theta\xi_{t+1}(1-\alpha)A_{t+1}, \quad [33]$$

$$1+r_{t+1}^n = (1-\tau_{t+1}^k)\alpha A_{t+1}, \quad [34]$$

and

$$\xi_t \equiv \frac{1-\gamma-\alpha(1-\tau_t^k)}{1-\alpha}. \quad [35]$$

Step 1. We first rewrite $\left\{ \left[E(\Gamma_{t+1}^{1-\eta}) \right]^{\frac{1}{1-\eta}} \right\}^{1-\sigma\eta} \left[E_t(1+r_{t+1}^n) \Gamma_{t+1}^{-\eta} \right]^{-\sigma}$ in the denominator of Eq. [32] as

$$\left[E(\Gamma_{t+1}^{1-\eta}) \right]^{\frac{1-\sigma\eta}{1-\eta}} \left[E_t(1+r_{t+1}^n) \Gamma_{t+1}^{-\eta} \right]^{-\sigma}. \quad [36]$$

From Eqs. [33-34] we have that

$$\Gamma_{t+1}^{1-\eta} = \left[(1+n+\theta)\Phi_{t+1} \right]^{1-\eta} A_{t+1}^{1-\eta}, \text{ and}$$

$$(1+r_{t+1}^n) \Gamma_{t+1}^{-\eta} = (1-\tau_{t+1}^k)\alpha \left[(1+n+\theta)\Phi_{t+1} \right]^{-\eta} A_{t+1}^{1-\eta},$$

where $\Phi_{t+1} \equiv (1-\tau_{t+1}^k)\alpha + \theta\xi_{t+1}(1-\alpha)/(1+n+\theta)$. Thus²⁵,

$$E(\Gamma_{t+1}^{1-\eta}) = \left[(1+n+\theta)\Phi_{t+1} \right]^{1-\eta} E(A_{t+1}^{1-\eta}), \text{ and}$$

$$E\left[(1+r_{t+1}^n) \Gamma_{t+1}^{-\eta} \right] = (1-\tau_{t+1}^k)\alpha \left[(1+n+\theta)\Phi_{t+1} \right]^{-\eta} E(A_{t+1}^{1-\eta}).$$

(25) Bearing in mind that if $\delta = 1$, τ_{t+1}^w is independent of A_{t+1} .

Therefore Eq. [36] can be rewritten as

$$[(1+n+\theta)\Phi_{t+1}] \times [(1-\tau_{t+1}^k)\alpha]^{-\sigma} [E(A_{t+1}^{1-\eta})]_{1-\eta}^{\frac{1-\sigma}{1-\eta}}. \quad [37]$$

Step 2. We next obtain that

$$E(A_{t+1}^{1-\eta}) = E[\hat{A}e^{\varepsilon_{t+1}}]^{1-\eta} = \hat{A}^{1-\eta} \exp\left[\frac{1}{2}\eta(\eta-1)\sigma_\varepsilon^2\right],$$

so that Eq. [37] can be rewritten as

$$[(1+n+\theta)\Phi_{t+1}] \times [(1-\tau_{t+1}^k)\alpha]^{-\sigma} \hat{A}^{1-\sigma} \exp\left[\frac{1}{2}\eta(\sigma-1)\sigma_\varepsilon^2\right]. \quad [38]$$

Step 3. Substitution of Eq. [38] into Eq. [32] yields, after rearranging terms,

$$1+g_t = \frac{\xi_t(1-\alpha)A_t}{(1+n+\theta)\left\{1+\Phi_{t+1}[\beta(1-\tau_{t+1}^k)\alpha]^{-\sigma}\left[e^{\frac{-\eta}{2}\sigma_\varepsilon^2}E(A_{t+1})\right]^{1-\sigma}\right\}},$$

where ξ_t is given in Eq. [35]. Finally, denoting $\tilde{A} = e^{\frac{-\eta}{2}\sigma_\varepsilon^2}\hat{A}$, where $\hat{A} = E(A_{t+1})$, yields the result.

• Proof of Proposition 4

Part i) Note that Ψ^S and Ψ^{NS} (defined in Eq. [17] and in footnote 14, respectively) are positive, $E(I_1^S)$ and I_1^{NS} are linear in \tilde{A} and \hat{A} , respectively, and $0 < \tilde{A} < \hat{A}$ if $\eta \times \sigma_\varepsilon^2 > 0$.

Part ii) Taking expectations on the absolute value of I_2^S in Eq. [22], rearranging terms and defining $Z \equiv \beta^{-\sigma}\hat{A}^{1-\sigma}\left\{[(1-\tau^k)\alpha]^{1-\sigma} + \Omega\theta\xi(1-\alpha)[(1-\tau^k)\alpha]^{-\sigma}\right\}$, one can see that $E(|I_2^S|) > |I_2^{NS}|$ [the latter defined in Eq. [25] if and only if

$$\left[e^{-\eta\sigma_\varepsilon^2(1-\sigma)/2} - 1\right]\left[1 - Z^2e^{-\eta\sigma_\varepsilon^2(1-\sigma)/2}\right] > 0,$$

or alternatively,

$$\left[e^{-\eta\sigma_\varepsilon^2(1-\sigma)/2} - 1\right]\left[1 - Ze^{-\eta\sigma_\varepsilon^2(1-\sigma)/4}\right] > 0,$$

where $\sigma \neq 1$ and $\eta\sigma_\varepsilon^2 > 0$.

Part iii) $E(I_3^S)$ and I_3^{NS} are trivially negative as $A_t > 0 \forall t$ and Ψ^S and Ψ^{NS} are positive. Finally, it is straightforward to show that the condition on the relative size of $E(|I_3^S|)$ and $|I_3^{NS}|$ coincides with that of the relative size of $E(|I_2^S|)$ and $|I_2^{NS}|$

• Proof of Proposition 8

From Eq. [15], and substituting $\xi_t = \frac{1-\gamma-\alpha(1-\tau^k)}{1-\alpha}$, we obtain

$$g_t = \frac{\Omega[1-\gamma-\alpha(1-\tau^k)]A_t}{\{1+\Phi_{t+1}[\beta(1-\tau^k)\alpha]^\sigma F\}} - 1, \quad [39]$$

where $\Omega \equiv (1+n+\theta)^{-1}$, $F \equiv \left[e^{\frac{-\eta}{2}\sigma_c^2} \hat{A} \right]^{1-\sigma}$, and $\Phi_{t+1} \equiv \alpha(1-\tau^k) + \theta\xi_{t+1}(1-\alpha)/(1+n+\theta)$.

Differentiating both sides of Eq. [39] with respect to τ^k yields

$$\begin{aligned} \left. \frac{\partial g_t}{\partial \tau^k} \right|_{\text{constant } \gamma} > 0 &\Leftrightarrow \alpha \left\{ 1 + \Phi_{t+1} [\beta(1-\tau^k)\alpha]^\sigma F \right\} \\ &+ \xi_t (1-\alpha) (\beta\alpha)^{-\sigma} F (1-\tau^k)^{-\sigma} \left[\frac{\alpha(1+n)}{1+n+\theta} - \frac{\Phi_{t+1}\sigma}{1-\tau^k} \right] > 0. \end{aligned}$$

$$\text{Thus, } \alpha(1+n)(1+n+\theta)^{-1} - \Phi_{t+1}\sigma(1-\tau^k)^{-1} \geq 0 \quad \Leftrightarrow \quad \sigma \leq \frac{\alpha(1+n)(1-\tau^k)}{(1+n+\theta)\Phi_{t+1}} =$$

$$\frac{\alpha(1+n)(1-\tau^k)}{(1+n+\theta)\alpha(1-\tau^k) + \theta\xi_{t+1}(1-\alpha)} = \frac{\alpha(1+n)(1-\tau^k)}{(1+n+\theta)\alpha(1-\tau^k) + \theta[1-\gamma-\alpha(1-\tau^k)]} =$$

$$\frac{\alpha(1+n)(1-\tau^k)}{(1+n)\alpha(1-\tau^k) + \theta(1-\gamma)} \text{ is sufficient for } \left. \frac{\partial g_t}{\partial \tau^k} \right|_{\text{constant } \gamma} > 0.$$



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RESUMEN

Analizamos los efectos de la imposición sobre las rentas del capital en el crecimiento a largo plazo en una economía de generaciones solapadas de dos periodos, y estocástica y con tecnología AK. Se distingue entre las rentas del capital y las rentas del trabajo, y entre las actitudes hacia el riesgo y la sustitución intertemporal del consumo. Mostramos las condiciones necesarias y suficientes bajo las cuales i) incrementos en la imposición sobre las rentas de capital dan lugar a mayores tasas de crecimiento, y ii) el efecto de variaciones en el tipo impositivo sobre la renta del capital en la tasa de crecimiento puede tener signos opuestos en economías estocásticas y en economías deterministas. Las simulaciones numéricas muestran que las economías deterministas de generaciones solapadas son una buena aproximación para el cálculo del efecto de los impuestos en la tasa de crecimiento de equilibrio en economías estocásticas al igual que en Smith (1996).

Palabras clave: crecimiento, imposición sobre rentas de capital, ahorro por motivo precaución.

Clasificación JEL: E21, H24, O49.

